

FUNCTIONAL ANALYSIS

ICTP - 2019

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PROBLEM SET 4

Problem 31. (Characterization of compactness via sequences) Let (X, d) be a metric space. Show that X is compact if and only if every sequence $\{x_n\}_{n \geq 1} \subset X$ admits a subsequence $\{x_{n_k}\}$ that converges to an element in X .

Problem 32. (Tychonoff's theorem) Let $\{X_\alpha\}_{\alpha \in I}$ be a family of compact topological spaces, indexed by some set I . Show that the product space

$$X = \prod_{\alpha \in I} X_\alpha$$

is compact in the product topology.

Problem 33. Brezis' book - exercise 3.4.

Problem 34. Brezis' book - exercise 3.5.

Problem 35. Brezis' book - exercise 3.8.

Problem 36. Brezis' book - exercise 3.16.

Problem 37. Brezis' book - exercise 3.17.

Problem 38. Brezis' book - exercise 3.18.

Problem 39. Brezis' book - exercise 3.24.

Problem 40. Brezis' book - exercise 3.25.

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